On Hsu-Structure Manifold, $\phi-$ Multirecurrent and Symmetric

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Abstract -- In this paper we have defined $\phi(12)$, $\phi(13)$, $\phi(23)$, Ricci $\phi(12)$-Multirecurrent and multirecurrent symmetric Hsu- Structure manifold. Furthermore theorems on above ϕ - Multirecurrent and Multirecurrent symmetric Hsu -Structure manifold involving equivalent conditions with respect to various curvature tensors have also been discussed

Index Terms- Multirecurrent parameter, Curvature Tensors, Hsu-structure manifold.

1. INTRODUCTION

If on an even dimensional manifold Vn, n = 2m of differentiability class C^∞, there exists a vector valued real linear function ϕ, satisfying

$$\phi^2 = a^r I_n, \tag{1.1a}$$

$$\overline{\overline{X}} = a^r X \text{, for arbitrary vector field X.} \tag{1.1b}$$

where $\overline{X} = \phi X$, 0 ≤ r ≤ n and 'a' is a real or imaginary number.

Then { ϕ } is said to give to V_n a Hsu-structure defined by the equations (1.1) and the manifold V_n is called a Hsu-structure manifold.

The equation (1.1) gives different structure for different values of 'a' and 'r'.
If $r = 0$, it is an almost product structure.
If $a = 0$, it is an almost tangent structure.
If $r = \pm 1$ and $a = +1$, it is an almost product structure.
If $r = \pm 1$ and $a = -1$, it is an almost complex structure.
If $r = 2$ then it is a GF-structure which includes
π-structure for $a \neq 0$,
an almost complex structure for $a = \pm i$,
an almost product structure for $a = \pm 1$,
an almost tangent structure for $a = 0$.

Let the Hsu-structure be endowed with a metric tensor g, Such that

$$g(\phi X, \phi Y) + a^r g(X, Y) = 0.$$

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Then $\{\phi, g\}$ is said to give to V_n - metric Hsu-structure and V_n is called a metric Hsu-structure manifold.

The curvature tensor K, a vector -valued tri-linear function w.r.t. the connexion D is given by

$$K(X,Y)Z = D_X D_Y Z - D_Y D_X Z - D_{[X,Y]} Z, \tag{1.2a}$$

where

$$[X,Y] = D_X Y - D_Y X. \tag{1.2b}$$

The Ricci tensor S in V_n is given by

$$S(Y,Z) = (C_1^1 K)(Y,Z). \tag{1.3}$$

Where by $(C_1^1 K)(Y,Z)$, we mean the contraction of $K(X,Y)Z$ with respect to first slot.

For Ricci tensor, we also have

$$S(Y,Z) = S(Z,Y), \tag{1.4}$$

A linear map γ defined by,

$$S(Y,Z) = g(\gamma(Y), Z) = g(Y, \gamma(Z)), \tag{1.5}$$

The scalar k define by $k = C_1^1 \gamma$ (1.6)

is called the scalar curvature of V_n.

Let W, C, L and V be the Projective, conformal, conharmonic and concircular curvature tensors respectively given by

$$W(X,Y)Z = K(X,Y)Z - \frac{1}{(n-1)}[S(Y,Z)X - S(X,Z)Y]. \tag{1.7}$$

$$C(X,Y)Z = K(X,Y)Z - \frac{1}{(n-2)}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)\gamma(X) - g(X,Z)\gamma(Y)] + \frac{k}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]. \tag{1.8}$$

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$$L(X,Y)Z = K(X,Y)Z - \frac{1}{(n-2)} \left[S(Y,Z)X - S(X,Z)Y - g(X,Z)\gamma(Y) + g(Y,Z)\gamma(X)\right]. \quad (1.9)$$

$$V(X,Y)Z = K(X,Y)Z - \frac{k}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]. \quad (1.10)$$

A manifold is said to be recurrent, if

$$(\nabla_{T_1} R)(X,Y)Z = A_1(T_1)R(X,Y)Z. \quad (1.11)$$

The recurrent manifold is said to be symmetric, if

$$A_1(T_1) = 0, \quad (1.12)$$

in the above equation.

A manifold is said to be birecurrent, if

$$(\nabla_{T_1}\nabla_{T_2} R)(X,Y)Z = A_2(T_1,T_2)R(X,Y)Z. \quad (1.13)$$

It is said to be birecurrent symmetric, if

$$A_2(T_1,T_2) = 0 \quad (1.14)$$

2. ϕ -MULTIRECURRENCE AND SYMMETRY OF DIFFERENT KINDS:

Let P , a vector- valued tri-linear function, be any one of the curvature tensor K,W,C,L or V then we will define multirecurrence of different kind as follows:**

Definition(2.1): A Hsu-structure manifold is said to be ϕ (12)-multirecurrent in P, if

$$a^r\left(\nabla_{T_n}\nabla_{T_{n-1}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} P\right)(X, \phi Y)Z$$

$$+\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} P\right)\left((\nabla_{T_n}\phi)\phi X, \phi Y\right)Z$$

$$+a^r\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} P\right)\left(X, (\nabla_{T_n}\phi)Y\right)Z$$

+..

$$+\left(\nabla_{T_n}\nabla_{T_{n-1}} \dots\dots\dots \nabla_{T_2} P\right)\left((\nabla_{T_1}\phi)\phi X, \phi Y\right)Z$$

$$+a^r\left(\nabla_{T_n}\nabla_{T_{n-1}} \dots\dots\dots \nabla_{T_2} P\right)\left(X, (\nabla_1\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_1} P\right)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_1} P\right)\left((\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right)Z$$

+..

$$+\left(\nabla_{T_n}\nabla_{T_{n-1}} \dots\dots\dots \nabla_{T_3} P\right)\left((\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right)Z$$

$$+\left(\nabla_{T_n}\nabla_{T_{n-1}} \dots\dots\dots \nabla_{T_3} P\right)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_2}\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_1} P\right)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, \phi Y\right)Z$$

$$+a^r\left(\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_1} P\right)\left(X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y\right)Z$$

+..

$$+\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_3} P\right)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right)Z$$

$$+a^r\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_3} P\right)\left(X, (\nabla_{T_2}\nabla_{T_1}\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-3}} \dots\dots\dots \nabla_{T_1} P\right)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-3}} \dots\dots\dots \nabla_{T_1} P\right)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)X, (\nabla_{T_n}\phi)Y\right)Z$$

+..

$$+\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_4} P\right)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_3}\nabla_{T_2}\phi)Y\right)Z$$

$$+\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_4} P\right)\left((\nabla_{T_3}\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-3}} \dots\dots \nabla_{T_1} P\right)\left((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)\phi X, \phi Y\right)Z$$

$$+a^r\left(\nabla_{T_{n-3}} \dots\dots \nabla_{T_1} P\right)\left(X, (\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y\right)Z$$

+..

$$+\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_4} P\right)\left((\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right)Z$$

$$+a^r\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_4} P\right)\left(X, (\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-4}} \dots\dots \nabla_{T_1} P\right)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-4}} \dots\dots \nabla_{T_1} P\right)\left((\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y\right)Z$$

+..

$$+\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} P\right)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X, (\nabla_{T_4}\nabla_{T_3}\phi)Y\right)Z$$

$$+\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} P\right)\left((\nabla_{T_4}\nabla_{T_3}\phi)\phi X, (\nabla_{T_2}\nabla_{T_1}\phi)Y\right)$$

$$+\left(\nabla_{T_{n-4}} \dots \nabla_{T_1} P\right)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y\right)Z$$

$$+\left(\nabla_{T_{n-4}} \dots \nabla_{T_1} P\right)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right)Z$$

+..

$$+\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} P\right)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)Y\right)Z$$

$$+\left(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} P\right)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right)Z$$

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$+\left(\nabla_{T_{n-4}}\ldots\ldots\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)\phi X,\phi Y\right)Z$

$+a^r\left(\nabla_{T_{n-4}}\ldots\ldots\ldots\nabla_{T_1}P\right)\left(X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_5}P\right)\left(\left(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi\right)\phi X,\phi Y\right)Z$

$+a^r\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_5}P\right)\left(X,\left(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi\right)Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+\left(\nabla_{T_n}P\right)\left(\left(\nabla_{T_{n-1}}\phi\right)(\phi X),\left(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}\right)(Y)\right)Z$

$+\left(\nabla_{T_n}P\right)\left(\left(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}\phi\right)\phi X,\left(\nabla_{T_{n-1}}\phi\right)Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+\left(\nabla_{T_1}P\right)\left(\left(\nabla_{T_2}\phi\right)\phi X,\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_3}\phi\right)Y\right)Z$

$+\left(\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_3}\phi\right)\phi X,\left(\nabla_{T_2}\phi\right)Y\right)Z$

$+P\left(\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_1}\phi\right)\phi X,\phi Y\right)Z$

$+a^rP\left(X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\ldots\nabla_{T_1}\phi\right)Y\right)Z$

$$=a^rA_n(T_1,T_2,\ldots\ldots\ldots\ldots\ldots T_n)P(X,\phi Y)Z. \tag{2.1a}$$

Or equivalently

$a^r\left(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\ldots\nabla_{T_2}\nabla_{T_1}P\right)(\phi X,Y)Z$

$+a^r\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_2}\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\phi\right)X,Y\right)Z$

$+\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_2}\nabla_{T_1}P\right)\left(\phi X,\left(\nabla_{T_n}\phi\right)\phi Y\right)Z$

$+a^r\left(\nabla_{T_n}\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_2}\nabla_{T_1}P\right)\left(\left(\nabla_{T_{n-1}}\phi\right)X,Y\right)Z$

$+\left(\nabla_{T_n}\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_2}\nabla_{T_1}P\right)\left(\phi X,\left(\nabla_{T_{n-1}}\phi\right)\phi Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+a^r\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_2}P\right)\left(\left(\nabla_{T_1}\phi\right)X,Y\right)Z$

$+\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_2}P\right)\left(\phi X,\left(\nabla_{T_1}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\phi\right)X,\left(\nabla_{T_{n-1}}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_{n-1}}\phi\right)X,\left(\nabla_{T_n}\phi\right)\phi Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+\left(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\ldots\nabla_{T_3}P\right)\left(\left(\nabla_{T_2}\phi\right)X,\left(\nabla_{T_1}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\ldots\nabla_{T_3}P\right)\left(\left(\nabla_{T_1}\phi\right)X,\left(\nabla_{T_2}\phi\right)\phi Y\right)Z$

$+a^r\left(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\phi\right)X,Y\right)Z$

$+\left(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}P\right)\left(\phi X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\phi\right)\phi Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+a^r\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_3}P\right)\left(\left(\nabla_{T_2}\nabla_{T_1}\phi\right)X,Y\right)Z$

$+\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_3}P\right)\left(\phi X,\left(\nabla_{T_2}\nabla_{T_1}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_{n-3}}\ldots\ldots\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\phi\right)X,\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_{n-3}}\ldots\ldots\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi\right)X,\left(\nabla_{T_n}\phi\right)\phi Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_4}P\right)\left(\left(\nabla_{T_1}\phi\right)X,\left(\nabla_{T_3}\nabla_{T_2}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_4}P\right)\left(\left(\nabla_{T_3}\nabla_{T_2}\phi\right)X,\left(\nabla_{T_1}\phi\right)\phi Y\right)Z$

$+a^r\left(\nabla_{T_{n-3}}\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi\right)X,Y\right)Z$

$+\left(\nabla_{T_{n-3}}\ldots\nabla_{T_1}P\right)\left(\phi X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi\right)\phi Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+a^r\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_4}P\right)\left(\left(\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi\right)X,Y\right)Z$

$+\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_4}P\right)\left(\phi X,\left(\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_{n-4}}\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\phi\right)X,\left(\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_{n-4}}\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\phi\right)\phi Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_5}P\right)\left(\left(\nabla_{T_2}\nabla_{T_1}\phi\right)X,\left(\nabla_{T_4}\nabla_{T_3}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_5}P\right)\left(\left(\nabla_{T_4}\nabla_{T_3}\phi\right)X,\left(\nabla_{T_2}\nabla_{T_1}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_{n-4}}\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_n}\phi\right)X,\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)\phi Y\right)Z$

$+\left(\nabla_{T_{n-4}}\ldots\nabla_{T_1}P\right)\left(\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)X,\left(\nabla_{T_n}\phi\right)\phi Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

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$+(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_5}P)\left((\nabla_{T_1}\phi)X,(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)\phi Y\right)Z$

$+(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_5}P)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)X,(\nabla_{T_1}\phi)\phi Y\right)Z$

$+a^r(\nabla_{T_{n-4}}\ldots\nabla_{T_1}P)\left((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)X,Y\right)Z$

$+(\nabla_{T_{n-4}}\ldots\nabla_{T_1}P)(\phi X,(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi Y)Z$

+...

$+a^r(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_5}P)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)X,Y\right)Z$

$+(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_5}P)(\phi X,(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi Y)Z$

+...

+...

$+(\nabla_{T_n}P)\left((\nabla_{T_{n-1}}\phi)X,(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}\phi)\phi Y\right)Z$

$+(\nabla_{T_n}P)\left((\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}\phi)X,(\nabla_{T_{n-1}}\phi)\phi Y\right)Z$

+...

$+(\nabla_{T_1}P)\left((\nabla_{T_2}\phi)X,(\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_3}\phi)\phi Y\right)Z$

$+(\nabla_{T_1}P)\left((\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_3}\phi)X,(\nabla_{T_2}\phi)\phi Y\right)Z$

$+a^rP\left((\nabla_{T_n}\ldots\ldots\ldots\nabla_{T_1}\phi)X,Y\right)Z$

$+P(\phi X,(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\ldots\nabla_{T_1}\phi)\phi Y)Z$

$$= a^rA_n(T_1,T_2,\ldots\ldots\ldots\ldots\ldots T_n)P(\phi X,Y)Z. \tag{2.1b}$$

Remark: Similarly we can define Ricci ϕ (12)-multirecurrent, ϕ (13) and ϕ (23)-multirecurrent in Hsu-structure manifold.

Note: The total no. of terms in the left hand side of equation (2.1a) & (2.1b) is

$${}^nC_0+{}^nC_1 2^1+{}^nC_2 2^2+\ldots\ldots\ldots+{}^nC_r 2^r$$

$$+\ldots\ldots\ldots+{}^nC_n 2^n=3^n$$

Where in ${}^nC_r$, n is the total no. of ∇ operators operated on the tensor P or Ricci and structure ϕ and $r=0,1,2\ldots\ldots n-1,n$ is the numbers of ∇ 's associated with the structure ϕ only.

Definition (2.2) : A ϕ (12), ϕ (13), ϕ (23), multirecurrent Hsu-structure manifold is said to be P-symmetric or Ricci-symmetric if

$$A_n(T_1,T_2,\ldots\ldots\ldots\ldots\ldots,T_n)=0. \tag{2.2}$$

Theorem (1.1): In a ϕ (12)-multirecurrent Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then third also holds:

a) it is conformal ϕ (12)-multirecurrent,

b) it is conharmonic ϕ (12)-multirecurrent,

c) it is concircular ϕ (12)-multirecurrent.

Proof: from equations (1.8), (1.9), (1.10) and applying ϕ on X and Y in the resulting equation, we have

$$C(\phi X,\phi Y)Z = L(\phi X,\phi Y)Z+\frac{n}{n-2}\{K(\phi X,\phi Y)Z-V(\phi X,\phi Y)Z\} \tag{2.3}$$

Multiplying the resulting equation obtained by the equations (1.8), (1.9), (1.10) by

$a^rA_n(T_1,T_2,\ldots\ldots,T_n)$, we get

$$a^rA_n(T_1,T_2,\ldots\ldots,T_n)C(X,\phi Y)Z = a^rA_n(T_1,T_2,\ldots\ldots,T_n)L(X,\phi Y)Z$$

$$+\frac{a^rA_n(T_1,T_2,\ldots\ldots,T_n)}{n-2}\{K(X,\phi Y)Z-V(X,\phi Y)Z\}. \tag{2.4}$$

Differentiating equation (2.3) with respect to $T_1,T_2,\ldots\ldots,T_n$ successively ,we get

$a^r(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\nabla_{T_2}\nabla_{T_1}C)(X,\phi Y)Z$

$+(\nabla_{T_{n-1}}\ldots\ldots\nabla_{T_2}\nabla_{T_1}C)\left((\nabla_{T_n}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_{n-1}}\ldots\ldots\nabla_{T_1}C)(X,(\nabla_{T_n}\phi)Y)Z$

+...

$+(\nabla_{T_n}\ldots\ldots\nabla_{T_2}C)\left((\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_n}\ldots\ldots\nabla_{T_2}C)(X,(\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-2}}\ldots\ldots\nabla_{T_1}C)\left((\nabla_{T_n}\phi)\phi X,(\nabla_{T_{n-1}}\phi)Y\right)Z$

$+(\nabla_{T_{n-2}}\ldots\ldots\nabla_{T_n}C)\left((\nabla_{T_n}\phi)\phi X,(\nabla_{T_n}\phi)Y\right)Z$

+...

$+(\nabla_{T_n}\ldots\ldots\nabla_{T_3}C)\left((\nabla_{T_1}\phi)\phi X,(\nabla_{T_2}\phi)Y\right)Z$

$+(\nabla_{T_n}\ldots\ldots\nabla_{T_3}C)\left((\nabla_{T_2}\phi)\phi X,(\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_{n-2}} \ldots\ldots\ldots . \nabla_{T_1} C)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_{n-2}} \ldots\ldots\ldots . \nabla_{T_1} C)(X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y)Z$

+...

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_3} C)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_3} C)(X, (\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-3}} \ldots\ldots\ldots . \nabla_{T_1} C)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y\right) Z$

$+(\nabla_{T_{n-3}} \ldots\ldots\ldots . \nabla_{T_1} C)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right) Z$

+...

$+(\nabla_n \ldots\ldots\ldots\ldots \nabla_4 C)((\nabla_{T_1})\phi X, (\nabla_{T_3}\nabla_{T_2}\phi)Y)Z$

$+(\nabla_n \ldots\ldots\ldots\ldots \nabla_4 C)((\nabla_{T_3}\nabla_{T_2})\phi X, (\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-3}} \ldots\ldots\ldots\ldots \nabla_{T_1} C)((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}})\phi X, \phi Y)Z$

$+a^r(\nabla_{T_{n-3}} \ldots\ldots\ldots\ldots \nabla_{T_1} C)(X, (\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y)Z$

+...

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_4} C)\left((\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_4} C)(X, (\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots . \nabla_{T_1} C)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_{n-2}}\nabla_{n-3}\phi)Y\right) Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots . \nabla_{T_1} C)\left((\nabla_{T_{n-2}}\nabla_{n-3}\phi)\phi X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y\right) Z$

+...

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_5} C)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X, (\nabla_{T_4}\nabla_{T_3}\phi)Y\right) Z$

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_5} C)\left((\nabla_{T_4}\nabla_{T_3}\phi)\phi X, (\nabla_{T_2}\nabla_{T_1}\phi)Y\right) Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots . \nabla_{T_1} C)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y\right) Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots . \nabla_{T_1} C)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right) Z$

+……………………………………….................

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_5} C)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)Y\right) Z$

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_5} C)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right) Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots . \nabla_{T_1} C)\left((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_{n-4}} \ldots\ldots\ldots . \nabla_{T_1} C)(X, (\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y)Z$

+……………………………………….................

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_5} C)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_5} C)(X, (\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+\cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

+...

$+(\nabla_{T_n} C)\left((\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_{n-2}} \ldots\ldots\ldots \nabla_{T_1}\phi)Y\right) Z$

$+(\nabla_{T_n} C)\left((\nabla_{T_{n-2}} \ldots\ldots\ldots \nabla_{T_1}\phi)\phi X, (\nabla_{T_{n-1}}\phi)Y\right) Z$

+...

$+(\nabla_{T_1} C)\left((\nabla_{T_2}\phi)\phi X, (\nabla_{T_n} \ldots\ldots \nabla_{T_3}\phi)Y\right) Z$

$+(\nabla_{T_1} C)\left((\nabla_{T_n} \ldots\ldots \nabla_{T_3}\phi)\phi X, (\nabla_{T_2}\phi)Y\right) Z$

$+C\left((\nabla_{T_n}\nabla_{T_{n-1}} \ldots\ldots \nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r C\left(X, (\nabla_{T_n}\nabla_{T_{n-1}} \ldots\ldots \nabla_{T_1}\phi)Y\right) Z$

$= a^r(\nabla_{T_n}\nabla_{T_{n-1}} \ldots\ldots\ldots . \nabla_{T_2}\nabla_{T_1} L)(X, \phi Y)Z$

$+(\nabla_{T_{n-1}} \ldots\ldots\ldots . \nabla_{T_2}\nabla_{T_1} L)\left((\nabla_{T_n}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_{n-1}} \ldots\ldots\ldots . \nabla_{T_1} L)(X, (\nabla_{T_n}\phi)Y)Z$

+...

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_2} L)\left((\nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_2} L)(X, (\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-2}} \ldots\ldots\ldots . \nabla_{T_1} L)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\phi)Y\right) Z$

$+(\nabla_{T_{n-2}} \ldots\ldots\ldots . \nabla_{T_1} L)\left((\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right) Z$

+...

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_3} L)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_2}\phi)Y\right) Z$

$+(\nabla_{T_n} \ldots\ldots\ldots . \nabla_{T_3} L)\left((\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right) Z$

$+(\nabla_{T_{n-2}} \ldots\ldots\ldots . \nabla_{T_1} L)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_{n-2}} \ldots\ldots\ldots . \nabla_{T_1} L)(X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y)Z$

+...

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_3}L\right)\left(\left(\nabla_{T_2}\nabla_{T_1}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_3}L\right)\left(X,\left(\nabla_{T_2}\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-3}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(\left(\nabla_{T_n}\phi\right)\phi X,\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-3}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi\right)\phi X,\left(\nabla_{T_n}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_4}L\right)\left(\left(\nabla_{T_1}\phi\right)\phi X,\left(\nabla_{T_3}\nabla_{T_2}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_4}L\right)\left(\left(\nabla_{T_3}\nabla_{T_2}\phi\right)\phi X,\left(\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-3}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_{n-3}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_4}L\right)\left(\left(\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_4}L\right)\left(X,\left(\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-4}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\phi\right)\phi X,\left(\nabla_{T_{n-2}}\nabla_{n-3}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-4}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(\left(\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)\phi X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_5}L\right)\left(\left(\nabla_{T_2}\nabla_{T_1}\phi\right)\phi X,\left(\nabla_{T_4}\nabla_{T_3}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_5}L\right)\left(\left(\nabla_{T_4}\nabla_{T_5}\phi\right)\phi X,\left(\nabla_{T_2}\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-4}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(\left(\nabla_{T_n}\phi\right)\phi X,\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-4}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(\left(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)\phi X,\left(\nabla_{T_n}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_5}L\right)\left(\left(\nabla_{T_1}\phi\right)\phi X,\left(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi\right)Y\right)Z$$

$$\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_5}L\right)\left(\left(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi\right)\phi X,\left(\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{n-4}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_{n-4}}\ldots\ldots\ldots.\nabla_{T_1}L\right)\left(X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_5}L\right)\left(\left(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_5}L\right)\left(X,\left(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi\right)Y\right)Z$$

$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}L\right)\left(\left(\nabla_{T_{n-1}}\phi\right)\phi X,\left(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_n}L\right)\left(\left(\nabla_{T_{n-2}}\ldots\ldots\ldots\nabla_{T_1}\phi\right)\phi X,\left(\nabla_{T_{n-1}}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_1}L\right)\left(\left(\nabla_{T_2}\phi\right)\phi X,\left(\nabla_{T_n}\ldots\ldots\nabla_{T_3}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_1}L\right)\left(\left(\nabla_{T_n}\ldots\ldots\nabla_{T_3}\phi\right)\phi X,\left(\nabla_{T_2}\phi\right)Y\right)Z$$

$$+L\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\nabla_{T_1}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^rL\left(X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\frac{n}{n-2}\{a^r\left(\nabla_{T_n}\nabla_{T_{n-1}}\ldots\ldots\ldots.\nabla_{T_2}\nabla_{T_1}K\right)(X,\phi Y)Z$$

$$+\left(\nabla_{T_{n-1}}\ldots\ldots\ldots.\nabla_{T_2}\nabla_{T_1}K\right)\left(\left(\nabla_{T_n}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_{n-1}}\ldots\ldots\ldots.\nabla_{T_1}K\right)\left(X,\left(\nabla_{T_n}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_2}K\right)\left(\left(\nabla_{T_1}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_2}K\right)\left(X,\left(\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-2}}\ldots\ldots\ldots.\nabla_{T_1}K\right)\left(\left(\nabla_{T_n}\phi\right)\phi X,\left(\nabla_{T_{n-1}}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-2}}\ldots\ldots\ldots.\nabla_{T_1}K\right)\left(\left(\nabla_{T_{n-1}}\phi\right)\phi X,\left(\nabla_{T_n}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_3}K\right)\left(\left(\nabla_{T_1}\phi\right)\phi X,\left(\nabla_{T_2}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_3}K\right)\left(\left(\nabla_{T_2}\phi\right)\phi X,\left(\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-2}}\ldots\ldots\ldots.\nabla_{T_1}K\right)\left(\left(\nabla_{T_n}\nabla_{T_{n-1}}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_{n-2}}\ldots\ldots\ldots.\nabla_{T_1}K\right)\left(X,\left(\nabla_{T_n}\nabla_{T_{n-1}}\phi\right)Y\right)Z$$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$$+\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_3}K\right)\left(\left(\nabla_{T_2}\nabla_{T_1}\phi\right)\phi X,\phi Y\right)Z$$

$$+a^r\left(\nabla_{T_n}\ldots\ldots\ldots.\nabla_{T_3}K\right)\left(X,\left(\nabla_{T_2}\nabla_{T_1}\phi\right)Y\right)Z$$

$$+\left(\nabla_{T_{n-3}}\ldots\ldots\ldots.\nabla_{T_3}K\right)\left(\left(\nabla_{T_n}\phi\right)\phi X,\left(\nabla_{T_{n-2}}\nabla_{T_{n-1}}\phi\right)Y\right)Z$$

$+(\nabla_{T_{n-3}} \ldots\ldots\ldots \nabla_{T_1}K)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)\phi X,(\nabla_{T_n}\phi)Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_4}K)\left((\nabla_{T_1}\phi)\phi X,(\nabla_{T_3}\nabla_{T_2}\phi)Y\right)Z$

$+(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_4}K)\left((\nabla_{T_3}\nabla_{T_2}\phi)\phi X,(\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_{n-3}} \ldots\ldots\ldots \nabla_{T_1}K)\left((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_n}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_{n-3}} \ldots\ldots\ldots \nabla_{T_1}K)(X,(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_4}K)\left((\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_4}K)(X,(\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots \nabla_{T_1}K)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X,(\nabla_{T_{n-2}}\nabla_{n-3}\phi)Y\right)Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots \nabla_{T_1}K)\left((\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X,(\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_5}K)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X,(\nabla_{T_4}\nabla_{T_3}\phi)Y\right)Z$

$+(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_5}K)\left((\nabla_{T_4}\nabla_{T_3}\phi)\phi X,(\nabla_{T_2}\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots \nabla_{T_1}K)\left((\nabla_{T_n}\phi)\phi X,(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y\right)Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots \nabla_{T_1}K)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X,(\nabla_{T_n}\phi)Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_5}K)\left((\nabla_{T_1}\phi)\phi X,(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)Y\right)Z$

$+(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_5}K)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)\phi X,(\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_{n-4}} \ldots\ldots\ldots \nabla_{T_1}K)\left((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_{n-4}} \ldots\ldots\ldots \nabla_{T_1}K)(X,(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_5}K)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_n \ldots\ldots\ldots \nabla_5 K)(X,(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+(\nabla_{T_n}K)\left((\nabla_{T_{n-1}}\phi)\phi X,(\nabla_{T_{n-2}} \ldots\ldots\ldots \nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_n}K)\left((\nabla_{T_{n-2}} \ldots\ldots\ldots \nabla_{T_1}\phi)\phi X,(\nabla_{T_{n-1}}\phi)Y\right)Z$

$+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$+(\nabla_{T_1}K)\left((\nabla_{T_2}\phi)\phi X,(\nabla_{T_n} \ldots\ldots \nabla_{T_3}\phi)Y\right)Z$

$+(\nabla_{T_1}K)\left((\nabla_{T_n} \ldots\ldots \nabla_{T_3}\phi)\phi X,(\nabla_{T_2}\phi)Y\right)Z$

$+K\left((\nabla_{T_n}\nabla_{T_{n-1}} \ldots\ldots \nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$+a^r K(X,(\nabla_{T_n}\nabla_{T_{n-1}} \ldots\ldots \nabla_{T_1}\phi)Y)Z$

$-a^r(\nabla_{T_n}\nabla_{T_{n-1}} \ldots\ldots\ldots \nabla_{T_2}\nabla_{T_1}V)(X,\phi Y)Z$

$-(\nabla_{T_{n-1}} \ldots\ldots\ldots \nabla_{T_2}\nabla_{T_1}V)\left((\nabla_{T_n}\phi)\phi X,\phi Y\right)Z$

$-a^r(\nabla_{T_{n-1}} \ldots\ldots\ldots \nabla_{T_1}V)(X,(\nabla_{T_n}\phi)Y)Z$

$-\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$-(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_2}V)\left((\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$-a^r(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_2}V)(X,(\nabla_{T_1}\phi)Y)Z$

$-(\nabla_{T_{n-2}} \ldots\ldots\ldots \nabla_{T_1}V)\left((\nabla_{T_n}\phi)\phi X,(\nabla_{T_{n-1}}\phi)Y\right)Z$

$-(\nabla_{T_{n-2}} \ldots\ldots\ldots \nabla_{T_1}V)\left((\nabla_{T_{n-1}}\phi)\phi X,(\nabla_{T_n}\phi)Y\right)Z$

$-\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$-(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_3}V)\left((\nabla_{T_1}\phi)\phi X,(\nabla_{T_2}\phi)Y\right)Z$

$-(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_3}V)\left((\nabla_{T_2}\phi)\phi X,(\nabla_{T_1}\phi)Y\right)Z$

$-(\nabla_{T_{n-2}} \ldots\ldots\ldots \nabla_{T_1}V)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X,\phi Y\right)Z$

$-a^r(\nabla_{T_{n-2}} \ldots\ldots\ldots \nabla_{T_1}V)(X,(\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y)Z$

$-\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$-(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_3}V)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$-a^r(\nabla_{T_n} \ldots\ldots\ldots \nabla_{T_3}V)(X,(\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$-(\nabla_{T_{n-3}} \ldots\ldots\ldots \nabla_{T_1}V)\left((\nabla_{T_n}\phi)\phi X,(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y\right)Z$

$-(\nabla_{T_{n-3}} \ldots\ldots\ldots \nabla_{T_1}V)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)\phi X,(\nabla_{T_n}\phi)Y\right)Z$

$-\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$-(\nabla_{T_n} \dots\dots\dots \nabla_{T_4} V)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_3}\nabla_{T_2}\phi)Y\right)Z$

$-(\nabla_{T_n} \dots\dots\dots \nabla_{T_4} V)\left((\nabla_{T_3}\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right)Z$

$-(\nabla_{T_{n-3}} \dots\dots\dots \nabla_{T_1} V)\left((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)\phi X, \phi Y\right)Z$

$-a^r(\nabla_{T_{n-3}} \dots\dots\dots \nabla_{T_1} V)(X, (\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y)Z$

$-\cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$-(\nabla_{T_n} \dots\dots\dots \nabla_{T_4} V)\left((\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right)Z$

$-a^r(\nabla_{T_n} \dots\dots\dots \nabla_{T_4} V)(X, (\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$-(\nabla_{T_{n-4}} \dots\dots\dots \nabla_{T_1} V)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y\right)Z$

$-(\nabla_{T_{n-4}} \dots\dots\dots \nabla_{T_1} V)\left((\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y\right)Z$

$-\cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$-(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} V)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X, (\nabla_{T_4}\nabla_{T_3}\phi)Y\right)Z$

$-(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} V)\left((\nabla_{T_4}\nabla_{T_3}\phi)\phi X, (\nabla_{T_2}\nabla_{T_1}\phi)Y\right)Z$

$-(\nabla_{T_{n-4}} \dots\dots\dots \nabla_{T_1} V)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y\right)Z$

$-(\nabla_{T_{n-4}} \dots\dots\dots \nabla_{T_1} V)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right)Z$

$-\cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$-(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} V)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)Y\right)Z$

$-(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} V)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right)Z$

$-(\nabla_{T_{n-4}} \dots\dots\dots \nabla_{T_1} V)\left((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X, \phi Y\right)Z$

$-a^r(\nabla_{T_{n-4}} \dots\dots\dots \nabla_{T_1} V)(X, (\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y)Z$

$-\cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$-(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} V)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right)Z$

$-a^r(\nabla_{T_n} \dots\dots\dots \nabla_{T_5} V)(X, (\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$-\cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$-\cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$-(\nabla_{T_n} V)\left((\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_1}\phi)Y\right)Z$

$-(\nabla_{T_n} V)\left((\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_1}\phi)\phi X, (\nabla_{T_{n-1}}\phi)Y\right)Z$

$-\cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$-(\nabla_{T_1} V)\left((\nabla_{T_2}\phi)\phi X, (\nabla_{T_n} \dots\dots \nabla_{T_3}\phi)Y\right)Z$

$-(\nabla_{T_1} V)\left((\nabla_{T_n} \dots\dots \nabla_{T_3}\phi)\phi X, (\nabla_{T_2}\phi)Y\right)Z$

$-V\left((\nabla_{T_n}\nabla_{T_{n-1}} \dots\dots \nabla_{T_1}\phi)\phi X, \phi Y\right)Z$

$$-a^r V(X, (\nabla_{T_n}\nabla_{T_{n-1}} \dots\dots \nabla_{T_1}\phi)Y)Z\}. \tag{2.5}$$

Subtracting equation (2.4) from equation (2.5) and then using the fact that ϕ (12)-multirecurrent Hsu-structure is conformal ϕ (12)-multirecurrent and conharmonic ϕ (12)-multirecurrent for the same recurrence parameter in the resulting equation, we get

$+a^r(\nabla_{T_n}\nabla_{T_{n-1}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} V)(X, \phi Y)Z$

$+(\nabla_{T_{n-1}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_n}\phi)\phi X, \phi Y\right)Z$

$+a^r(\nabla_{T_{n-1}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} V)(X, (\nabla_{T_n}\phi)Y)Z$

$+\cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$+(\nabla_{T_n} \dots\dots\dots \nabla_{T_2} V)\left((\nabla_{T_1}\phi)\phi X, \phi Y\right)Z$

$+a^r(\nabla_{T_n} \dots\dots\dots \nabla_{T_2} V)(X, (\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\phi)Y\right)Z$

$+(\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right)Z$

$+\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$+(\nabla_{T_n} \dots\dots\dots \nabla_{T_3} V)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_2}\phi)Y\right)Z$

$+(\nabla_{T_n} \dots\dots\dots \nabla_{T_3} V)\left((\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_1} V)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, Y\right)Z$

$+a^r(\nabla_{T_{n-2}} \dots\dots\dots \nabla_{T_1} V)(X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y)Z$

$+\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$+(\nabla_{T_n} \dots\dots\dots \nabla_{T_3} V)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right)Z$

$+a^r(\nabla_{T_n} \dots\dots\dots \nabla_{T_3} V)(X, (\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-3}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y\right)Z$

$+(\nabla_{T_{n-3}} \dots\dots\dots \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right)Z$

$+\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_4} V)\left((\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_n} \dots\dots\dots. \nabla_{T_4} V)(X, (\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-4}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y\right) Z$

$+(\nabla_{T_{n-4}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y\right) Z$

+..

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_5} V)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X, (\nabla_{T_4}\nabla_{T_3}\phi)Y\right) Z$

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_5} V)\left((\nabla_{T_4}\nabla_{T_3}\phi)\phi X, (\nabla_{T_2}\nabla_{T_1}\phi)Y\right) Z$

$+(\nabla_{T_{n-4}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-3}}\nabla_{T_{n-2}}\nabla_{T_{n-1}}\phi)Y\right) Z$

$+(\nabla_{T_{n-4}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_{n-3}}\nabla_{T_{n-2}}\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right) Z$

+..

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_5} V)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)Y\right) Z$

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_5} V)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right) Z$

$+(\nabla_{T_{n-4}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} V)\left((\nabla_{T_{n-3}}\nabla_{T_{n-2}}\nabla_{T_{n-1}}\nabla_{T_n}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_{n-4}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} V)(X, (\nabla_{T_{n-3}}\nabla_{T_{n-2}}\nabla_{T_{n-1}}\nabla_{T_n}\phi)Y)Z$

+..

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_5} V)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_n} \dots\dots\dots. \nabla_{T_5} V)(X, (\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+ \cdots \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

+..

$+(\nabla_{T_n} V)\left((\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_{n-2}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1}\phi)Y\right) Z$

$+(\nabla_{T_n} V)\left((\nabla_{T_{n-2}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1}\phi)\phi X, (\nabla_{T_{n-1}}\phi)Y\right) Z$

+..

$+(\nabla_{T_1} V)\left((\nabla_{T_2}\phi)\phi X, (\nabla_{T_n} \dots\dots\dots. \nabla_{T_3}\phi)Y\right) Z$

$+(\nabla_{T_1} V)\left((\nabla_{T_n} \dots\dots\dots. \nabla_{T_3}\phi)\phi X, (\nabla_{T_2}\phi)Y\right) Z$

$+V\left((\nabla_{T_n} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r V(X, (\nabla_{T_n} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$= a^r A_n(T_1, T_2, \dots\dots\dots, T_n) V(X, \phi Y)Z.$

which shows that the manifold is concircular ϕ (12)-multirecurrent.

Similarly, it can be shown that if a ϕ (12)-multirecurrent Hsu-structure manifold is conformal ϕ (12)-multirecurrent and concircular ϕ (12)-multirecurrent or conharmonic ϕ (12)-multirecurrent and concircular ϕ (12)-multirecurrent then it is either conharmonic ϕ (12)-multirecurrent or conformal-ϕ (12)-multirecurrent for the same recurrence parameter.

Theorem (1.2): In a ϕ (12)-multirecurrent symmetric Hsu-structure manifold, if any two of the following conditions hold for the same recurrence parameter, then third also holds:

a) it is conformal ϕ (12)-multirecurrent symmetric,

b) it is conharmonic ϕ (12)-multirecurrent symmetric,

c) it is concircular ϕ (12)-multirecurrentsymmetric.

Proof: Let a ϕ (12)-multirecurrent symmetric Hsu-structure manifold is conharmonic ϕ (12)-multirecurrent symmetric and concircular ϕ (12)-multirecurrent symmetric then from equation (2.5), we have

$a^r(\nabla_{T_n} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} C)(X, \phi Y)Z$

$+(\nabla_{T_{n-1}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} C)\left((\nabla_{T_n}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_{n-1}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} C)(X, (\nabla_{T_n}\phi)Y)Z$

+..

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_2} C)\left((\nabla_{T_1}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_n} \dots\dots\dots. \nabla_{T_2} C)(X, (\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-2}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} C)\left((\nabla_{T_n}\phi)\phi X, (\nabla_{T_{n-1}}\phi)Y\right) Z$

$+(\nabla_{T_{n-2}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} C)\left((\nabla_{T_{n-1}}\phi)\phi X, (\nabla_{T_n}\phi)Y\right) Z$

+..

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_3} C)\left((\nabla_{T_1}\phi)\phi X, (\nabla_{T_2}\phi)Y\right) Z$

$+(\nabla_{T_n} \dots\dots\dots. \nabla_{T_3} C)\left((\nabla_{T_2}\phi)\phi X, (\nabla_{T_1}\phi)Y\right) Z$

$+(\nabla_{T_{n-2}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} C)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X, \phi Y\right) Z$

$+a^r(\nabla_{T_{n-2}} \dots\dots\dots. \nabla_{T_2}\nabla_{T_1} C)(X, (\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y)Z$

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+...

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_3}C)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_n}\dots\dots\dots.\nabla_{T_3}C)(X,(\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-3}}\dots\dots\dots.\nabla_{T_1}C)\left((\nabla_{T_n}\phi)\phi X,(\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y\right)Z$

$+(\nabla_{T_{n-3}}\dots\dots\dots.\nabla_{T_1}C)\left((\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)\phi X,(\nabla_{T_n}\phi)Y\right)Z$

+...

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_4}C)\left((\nabla_{T_1}\phi)\phi X,(\nabla_{T_3}\nabla_{T_2}\phi)Y\right)Z$

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_4}C)\left((\nabla_{T_3}\nabla_{T_2}\phi)\phi X,(\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_{n-3}}\dots\dots\dots.\nabla_{T_1}C)\left((\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_{n-3}}\dots\dots\dots.\nabla_{T_1}C)(X,(\nabla_{T_n}\nabla_{T_{n-1}}\nabla_{T_{n-2}}\phi)Y)Z$

+...

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_4}C)\left((\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_n}\dots\dots\dots.\nabla_{T_4}C)(X,(\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+(\nabla_{T_{n-4}}\dots\dots\dots.\nabla_{T_1}C)\left((\nabla_{T_n}\nabla_{T_{n-1}}\phi)\phi X,(\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)Y\right)Z$

$+(\nabla_{T_{n-4}}\dots\dots\dots.\nabla_{T_1}C)\left((\nabla_{T_{n-2}}\nabla_{T_{n-3}}\phi)\phi X,(\nabla_{T_n}\nabla_{T_{n-1}}\phi)Y\right)Z$

+...

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_5}C)\left((\nabla_{T_2}\nabla_{T_1}\phi)\phi X,(\nabla_{T_4}\nabla_{T_3}\phi)Y\right)Z$

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_5}C)\left((\nabla_{T_4}\nabla_{T_3}\phi)\phi X,(\nabla_{T_2}\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_{n-4}}\dots\dots\dots.\nabla_{T_1}C)\left((\nabla_{T_n}\phi)\phi X,(\nabla_{T_{n-3}}\nabla_{T_{n-2}}\nabla_{T_{n-1}}\phi)Y\right)Z$

$+(\nabla_{T_{n-4}}\dots\dots\dots.\nabla_{T_1}C)(,(\nabla_{T_{n-3}}\nabla_{T_{n-2}}\nabla_{T_{n-1}}\phi)\phi X,(\nabla_{T_n}\phi)Y)Z$

+...

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_5}C)\left((\nabla_{T_1}\phi)\phi X,(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)Y\right)Z$

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_5}C)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\phi)\phi X,(\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_{n-4}}\dots\dots\dots.\nabla_{T_1}C)\left((\nabla_{T_{n-3}}\nabla_{T_{n-2}}\nabla_{T_{n-1}}\nabla_{T_n}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_{n-4}}\dots\dots\dots.\nabla_{T_1}C)(X,(\nabla_{T_{n-3}}\nabla_{T_{n-2}}\nabla_{T_{n-1}}\nabla_{T_n}\phi)Y)Z$

+...

$+(\nabla_{T_n}\dots\dots\dots.\nabla_{T_5}C)\left((\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$+a^r(\nabla_{T_n}\dots\dots\dots.\nabla_{T_5}C)(X,(\nabla_{T_4}\nabla_{T_3}\nabla_{T_2}\nabla_{T_1}\phi)Y)Z$

$+\cdots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

+...

$+(\nabla_{T_n}C)\left((\nabla_{T_{n-1}}\phi)\phi X,(\nabla_{T_{n-2}}\dots\dots\dots.\nabla_{T_1}\phi)Y\right)Z$

$+(\nabla_{T_n}C)\left((\nabla_{T_{n-2}}\dots\dots\dots.\nabla_{T_1}\phi)\phi X,(\nabla_{T_{n-1}}\phi)Y\right)Z$

$+\cdots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$

$+(\nabla_{T_1}C)\left((\nabla_{T_2}\phi)\phi X,(\nabla_{T_n}\dots\dots\dots.\nabla_{T_3}\phi)Y\right)Z$

$+(\nabla_{T_1}C)\left((\nabla_{T_n}\dots\dots\dots.\nabla_{T_3}\phi)\phi X,(\nabla_{T_2}\phi)Y\right)Z$

$+C\left((\nabla_{T_n}\dots\dots\dots.\nabla_{T_1}\phi)\phi X,\phi Y\right)Z$

$+a^rC(X,(\nabla_{T_n}\dots\dots\dots.\nabla_{T_1}\phi)Y)Z=0.$

Which shows that the manifold is conformal ϕ (12)-multirecurrent symmetric.

Similarly, it can be shown that if a manifold is either conharmonic ϕ (12)-multirecurrent symmetric and conformal ϕ (12)-multirecurrent symmetric or concircular ϕ (12)-multirecurrent symmetric and conformal ϕ (12)-multirecurrent symmetric then it is either concircular ϕ (12)-multirecurrent symmetric or conharmonic ϕ (12)-multirecurrent symmetric for the same recurrence parameter.

Note: Theorems of the type (1.1) and (1.2) can also be proved by taking Ricci ϕ (12) , ϕ (13) or ϕ (23)-multirecurrent and Ricci ϕ (12), ϕ (13) or ϕ (23)-multirecurrent symmetric Hsu-structure manifold.

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